Matrix Architecture: 3D-Printed and Simulated Kirigami Matrices & Auxetic Materials

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ABSTRACT

This paper explores the possibilities of kirigami geometry — folding with the addition of strategically placed cuts and holes — through simulation and kinetic and adaptive architectural assemblies. Typical kinetic assemblies consist of rigid components connected by mechanical joints that offer limited range of motion and tend to require mechatronic actuation. While mechanical motion is adequate for specific applications, mechanically motile systems lack the adaptive potential, elasticity, and embedded intelligence of adaptive structures. We propose to focus on the design of flexible matrices as a way of moving away from stiff, mechanical unitized systems and toward pliable, continuous 2D and 3D structures that can elastically change geometry in response to external stimuli without the need for external mechatronic energy input. As a proof-of-concept, we have produced an integrated panel-and-hinge assembly in which the panels and hinges are not discrete, mechanically connected components, but are instead functional zones of a continuous matrix. In addition, by controlling aspects of the individual units (panel size, hinge geometry, spacing, unit shape), we can induce larger-scale behavioral changes in the whole matrix.

Author Keywords

adaptive architecture; kirigami; simulation and modeling; computational design; 3D printing; material design; programmable matter; auxetic materials

1   INTRODUCTION

As part of two projects funded by the National Science Foundation in the Sabin Design Lab at Cornell University titled, eSkin and Kirigami in Architecture, Technology, and Science (KATS), this paper is one product of ongoing trans-disciplinary research spanning across the fields of architecture, cell biology, materials science, physics, electrical and systems engineering, and computer science. This paper explores the possibilities of kirigami in kinetic and adaptive architectural assemblies. Kirigami is similar to origami, but includes the addition of cuts and holes. The origin of the word comes from the Japanese kiru, "to cut," a geometric method and process that brings an extra, previously unattainable level of design, dynamics, and deployability to self-folding and -unfolding materials from the molecular to the architectural scale.

ColorFolds, a project produced by Sabin Design Lab 2014-2015, is our largest deployable structure generated with kirigami geometry. The assembly responds in a controlled, kinetic mode to contextual feedback, expressed through optical color and transparency change. The generative design process for ColorFolds began with an examination and study of kirigami processes as a means of creating doubly-curved surfaces through a simple implementation of gradient folding conditions. ColorFolds is an interactive folded assembly prototype composed of a lightweight, tessellated array of interactive components that fold and unfold in the presence or absence of people. ColorFolds was a successful prototype in the exploration of kirigami geometry and form, but the design relied heavily upon mechanical hinges. Shown in 1, complex mechatronic systems including linear actuators and several mechanical devices are integrated to actuate folding response to environmental input. This is essentially allocating almost all of the response control to fragile, error-prone mechatronic elements. A primary issue concerns scale: in order to effectively scale kirigami inspired foldable material assemblies like ColorFolds into deployable building materials, mechatronic systems are problematic. They are too fragile, maintenance and energy-heavy, and costly to support large-scale assemblies of folding panels. To address this issue, we have directed our research toward panel-and-hinge assemblies that integrate the hinge and panel as one composite in order to both decrease reliance on mechatronics and electrical energy for actuation and strive toward a
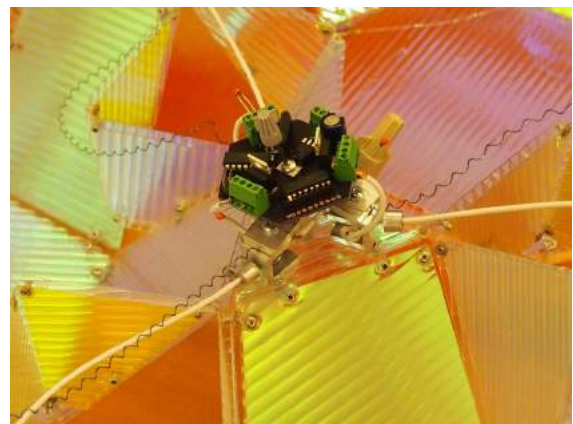

Figure 1. Mechatronic actuator from Sabin Design Lab's 2014-2015 project *Colorfolds.*

programmable material capable of controlled, elastic response to stimuli.

To further our research on kirigami with our findings from ColorFolds, our goal is threefold: 1) Create a flexible assembly that is not composed of rigid parts, but instead integrated and elastic, 2) Simulate the flexible assembly at a larger scale to understand its global behavior and 3) Create a customized pipeline enmeshed with material feedback between simulation, computational analysis, and the creation of new physical tests. These goals produce findings that inform our testing and representation of open, deployable and scalable structural elements and structures [1]. This paper describes the methods of our 3 step experimental loop: 1) design and prototyping of kirigami material composites [2, 3]. FEA verification with physical testing and 3) larger scale kirigami simulation [4,5]. The results contain relevant images of the physical models, experiments, and simulations of our process.

2 MATERIALS AND METHODS

2.1 Computational Model Design

The primary aim of our computational model is to design patterns with auxetic properties. Auxetic materials are structures with negative poisson ratio. That is, when tension is applied on the pattern, the model becomes thicker and stronger, unlike conventional materials that stretch and weaken with the addition of tension forces. To achieve this, the kirigami pattern consists of two parts: slit cuts and hinges, each defined by a series of tunable parameters. The slits are described by vertical length and the separation between the edge of one slit and the center of the adjacent slit. By changing these parameters, a series of flat sheets with auxetic perforations are created. The geometry is illustrated in Figure 2 (as developed in a research paper by Spencer Magleby of Brigham Young University).

To achieve auxetic properties, every square needs to rotate concurrently. In the physical tests, we found the connection between squares prone to rupture as this is where stress is

concentrated, causing failure in rotation. Thus, spring-like hinges are designed to enhance the strength of the linkages. The geometrical shape of the hinge and the width of it are flexible. The mechanical properties that determine the auxetic performance of the model, such as Young's Modulus and Poisson's ratio, can be modified through changing the shape and width in the hinge geometry. Finite element analysis (FEA) is performed on a single hinge to optimize the physical behavior and auxetic performance of the model. In addition to the stretching and compression force, the hinge also undertakes bending force when deformed to approximate a curved surface. When the hinge behaves in a bending-dominated mode of deformation, its thickness and width are key factors in determining the strength of the linkage, as can be seen in Figure 3 (as first conceptualized by Cho et al. in 2014 [2]).
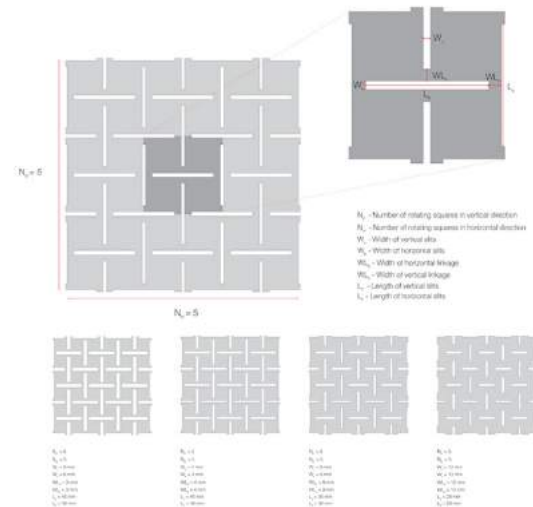

Figure 2. Parameters of slits within the kirigami model that provide auxetic characteristics.
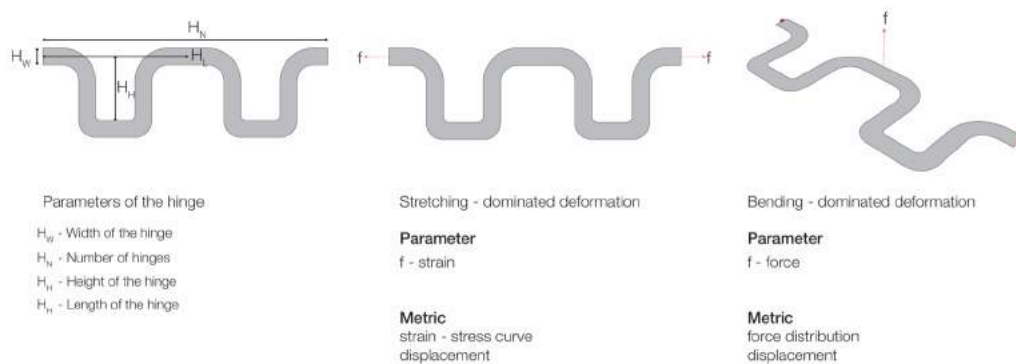

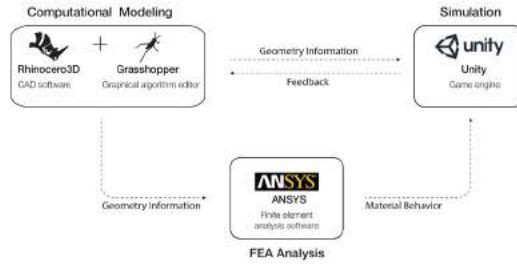**Figure 3.** Parameters of hinge parts in kirigami matrices.

Figure 4. Work-flow diagram depicting key tools and dialogue between programs.

The computational model is generated in Grasshopper for Rhinoceros, a visual programming language run within Rhinoceros 3D computer-aided design (CAD) application. Next, FEA is performed by engineering software, ANSYS, to simulate ideal material properties and to verify experimental results of the models. ANSYS is an engineering simulation software commonly used in industries to predict the life cycle and to simulate the performance of a product. For example, mechanical engineers may use ANSYS to simulate the strength of car structures before the physical crash tests, in order to save time and money in the design process. By changing parameters of slits, hinges and rotation of squares in Grasshopper, a series of 3D models can be created, as can be seen in Figure 3. These are concurrently simulated and analyzed via the connections between Rhino, Unity3D, a customizable graphics platform with advanced physics engine, and ANSYS for finite element analysis. The workflow diagram showing the coordination between different software can be simplified and visualized in Figure 4.

Material Prototyping
To test at the material scale, physical models are 3D printed out of ABS plastic and then cast into a thin sheet of silicone to form composite assemblies. The plastic models are meant to be geometrically flexible – variegated geometries enable stretching and buckling, even when printed in a rigid material like ABS plastic. The ABS plastic sheets are composed of two zones: a panel zone and a hinge zone. The panel and hinge are materially identical and are printed simultaneously, with the goal of integrating the panel and hinge into one continuous system. The ABS sheets are cast into shallow silicone sheets after they are printed, where the process is shown in Figure 5.

2.2 Scaled Unity3D Simulation
Biaxial stretching of the sheets is then simulated in Unity3D. Unity3D is typically used as a game engine. However, it is also used in other fields for its rendering and physics simulation capabilities. Abstractions of the kirigami sheets were created in Unity as rigid bodies. Rigid bodies are the class in Unity that allow objects in the simulated environment to react to physical forces. Performing simple

tests on these computational models allows us to understand the global behavior of our kirigami matrix.

First, a script was written to produce a grid of two-dimensional springs connected by Unity joints according to the slit parameters predetermined by our kirigami model. The Unity joint class connects two rigid bodies and behaves as a physical hinge. Pulling and twisting tests were performed on varying grid sizes to mimic our physical tests. The pulling experiments, involve setting the top row as anchor points and applying gravity to the remaining cells. These two-dimensional simulations give us insights into how the lattice cuts respond to simple manipulations on a larger scale.

Next, a script was written to produce a grid of three-dimensional rigid bodies. Halved spheres were included at hinge locations to represent the physical hinges. Moving to three dimensions and adding space between panels allowed for bending and compression tests. Active bending tests were performed by anchoring the bottom row highlighted by the red bounding box in test A of Figure 6. Gravity is then applied to the remaining cells. Tension tests were performed by anchoring the top row in test B of Figure 6 and applying gravity to the remaining cells.
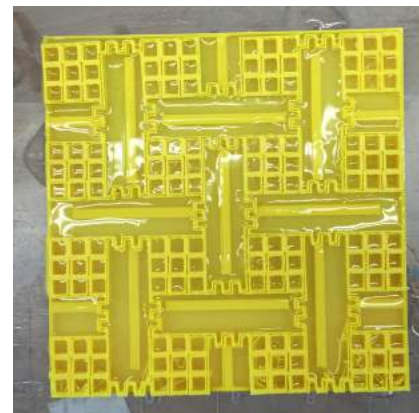

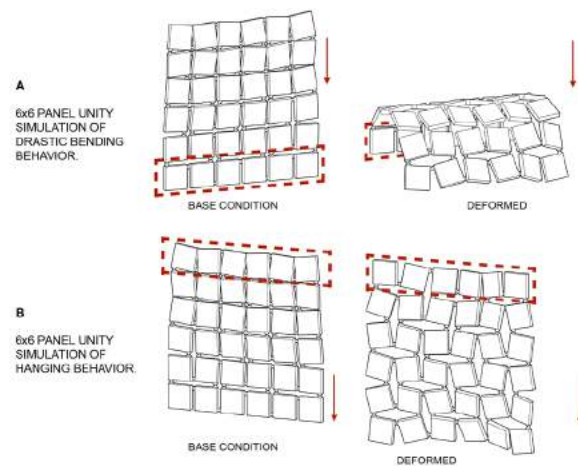**Figure 5.** Cast of ABS matrices into shallow silicone sheets.

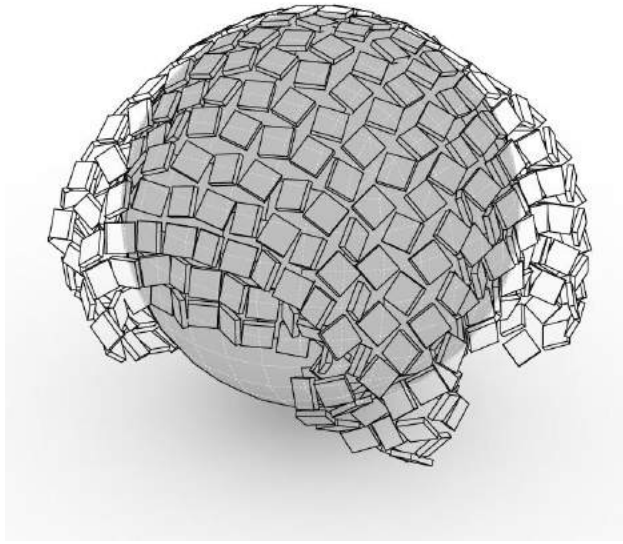**Figure 6.** Bending Test A and Stretching Test B.

Figure 7. A simulated kirigami sheet measuring 20x20 units, draped over a sphere in Unity.

Unity's lightweight physics engine makes it possible to drape kirigami sheets over simple geometries like the one shown in Figure 7. Stiffness of the hinges can be changed to test how more rigid sheets behave when deformed.

2.3 Experimental Loop

In order to optimize the hinge geometry in the physical design process, an experimental loop was developed. To constrain the variables in optimization, three different models with different quantities of hinges were designed and printed. FEA was performed on the three models with the use of ANSYS software. To verify the FEA results, stress and strain tests were performed using a Vernier Force Sensor and the Young's Modulus Graphs were generated with comparisons between the results of physical testing and FEA. After verifying the results, Young's modulus data were compared between the three models and optimized as a controllable attribute of the panel. Finally, the optimization results were input into Unity for large-scale structures design. In turn, a feedback loop is generated for the next round of geometry and material design.

ANSYS performs finite element analysis by creating meshes on our CAD model geometry. After defining the boundary conditions such as the applied forces and fixed points on the model, rigorous mathematical calculations were done on the edges of every mesh. The calculation continues until the numerical solutions on the meshes converge. Finally, the software post-processes the mathematical solutions into clearly visualized graphs and 3D contour plots. The user-friendly interface of ANSYS allows various definitions of boundary conditions. Loads, edge-fixed supports, and face-fixed supports were set for the static structural loading condition. Von-Mises stress, directional and total deformation, strain, and safety factor were selected for the output of the results. Von-Mises stress is a particular engineering stress that predicts the yielding conditions of materials, and safety factor determines if the material will reach the failure point when experiencing loads. While Von-Mises stress and strain were used to calculate Young's modulus of the model, directional and total deformation, as well as safety factor, enable in depth understanding of the material properties.

Then, physical experiments were further designed to compare the numerical results from previous physical tests with FEA results. The designs of the physical experiments were critical in constraining the variables and obtaining quantitative data. Two different experiments were set up for the validation of FEA - stretch test, and compression test.

Stretch test is set up as in Figure 8.A. The model is pinned at the corners on a 2D horizontal plane and forces were applied on the opposite (left) end with a 3D-printed jig in order to distribute the force evenly. A Vernier Dual-Range force sensor was attached to the force-applying end, and a camera was set up above the model. Certain force quantities were applied with the documentation of the force sensor, and the camera records the deformation of the model.

The compression test features 3D printed jigs in the setup as shown in Figure 8.B. The material composite was put inside the 3D printed jig so that the bottom surface of the model is supported and forces were applied on the top of the model. The 3D printed jigs were designed to constrain the model in a 2D plane. Quantitative forces were applied with the use of weights, and a camera was set up to record the results as in the stretch test.
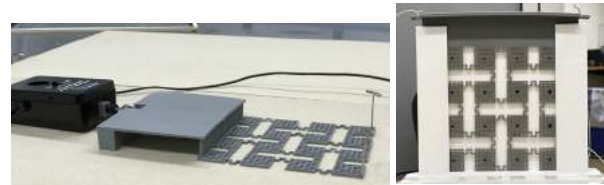

Figure 8. A. Stretch test setup (left). B. Compression test setup (right).

To post process the experimental results, camera images with different stresses were layered, shown in Figure 9. Centers of the local grids were marked, and strain is calculated by measuring the location of the marked center. Nominal strain in x, y directions were then derived from the universal engineering strain formula:

$$\varepsilon_{xx}^{[i,j]} = \frac{x^{(i+,,j)} - x^{(i,j)} + x^{(i+,,j+,)} - x^{(i,j+,)} - 2L_2}{2L_2}$$

$$\varepsilon_{yy}^{[i,j]} = \frac{x^{(i,j+,)} - x^{(i,j)} + x^{(i+,,j+,)} - x^{(i+,,j)} - 2L_2}{2L_2}$$

Stress was obtained by dividing the force sensor data with the surface area of the model. Young's modulus was then computed by taking the ratio of various stress and strain.
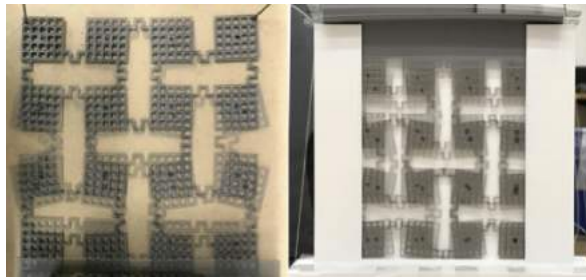
Figure 9. Post processing method for stretch test (left), compression test (right).

3   RESULTS

FEA were performed on a single hinge for studying the mechanical behaviors, avoiding geometric failures, and optimizing auxetic properties. The results are shown in Figure 10. The spring-like geometry was then determined to maximize the auxetic properties, where corner fillets avoid stress concentration. Then the sheet geometries were determined as in Figure 11.
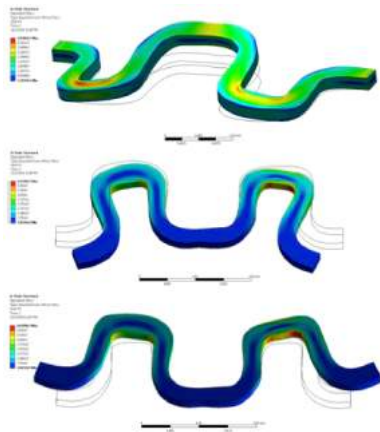

Figure 10. Von-Mises stress contours on a single hinge in FEA. Top to bottom: tension, compression, bending.
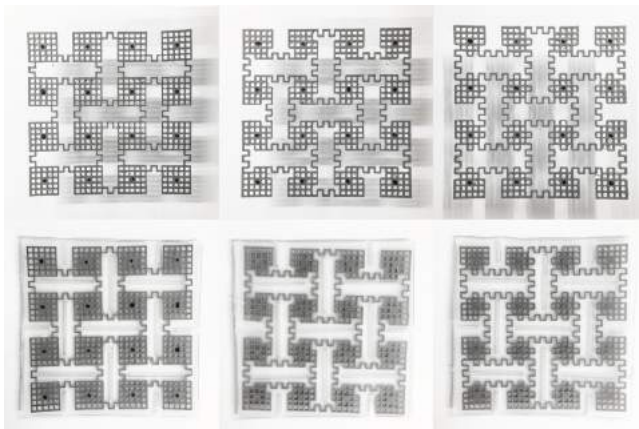

Figure 11. Final ABS and silicone matrices, differentiated by the number of grid units, the hinge travels into the adjacent panel. Across Row 1, Left to Right: 0 unit hinge, 1 unit hinge (on either side), 2 unit hinge (on either side). Bottom row: corresponding panels cast into silicone.
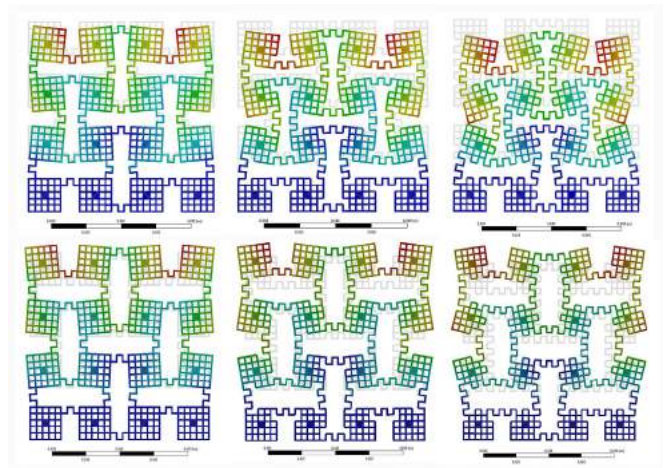

Figure 12. FEA results of tension test (upper row) and compression test (lower row). Non-deformed models were also included in light gray, behind the false-color results.
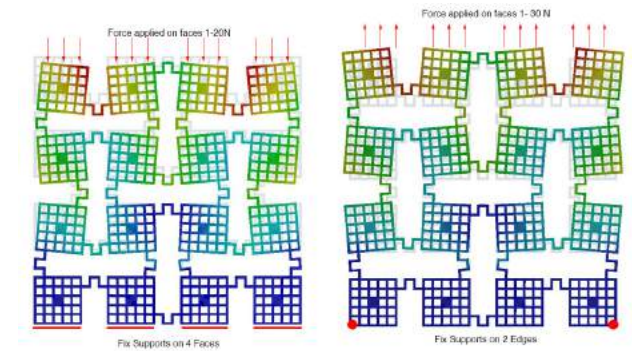

Figure 13. FEA Compression Test Setup (left), FEA Tension Test Setup (Right),

FEA were performed on both tension and compression tests, and visualized results can be seen in Figure 12. The setup for FEA can be seen in Figure 13. In tension tests, fixed supports were applied to the two corners to simulate the pins in the experiment. Force is a variable ranging from 1N to 30N applied to the 4 faces on the opposite end. In compression test, fixed supports were applied to the 4 faces on one end while the force is a variable ranging from 1N to 20N on the 4 faces on the other end. In both experiments, directional deformations were extracted with probes, and post-processed into local strain. Stresses were extracted as Von-Mises engineering stress.

FEA results demonstrate correct verification and validation with the experimental results. In verification, FEA results and experimental results, both included in Figure 14, show similar behavior in both tension and compression tests. The small deviations between the two datasets are possible effects from uncontrollable friction and gravity. To validate the results, the behavior of stress strain curves is consistent with actual material properties with an ultimate tensile stress (UTS) and a failure point. Variations in hinges show huge effects on the Young's modulus. Young's Modulus data evaluated at UTS in both compression test and tension test

are on the same magnitude with a standard deviation of 5.12. Hinge 0 has a Young's Modulus of 1.04 MPa, hinge 1 of 0.53 MPa, and hinge 2 of 0.454 MPa.
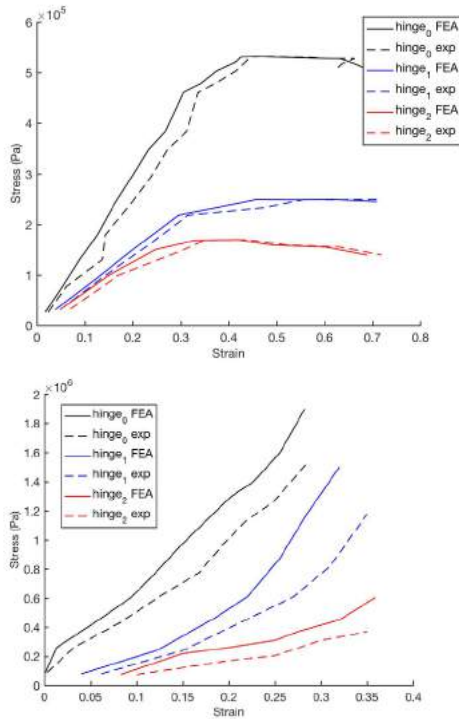

Figure 14. Stress Strain Curve of PLA Model From Stretch Test (top) and compression test (bottom).

4 DISCUSSION

The stress strain curves in the experimental loop show how the hinges affect Young's Modulus, and how the material properties were modified by the kirigami geometry. By comparing the data from the three different models, it is observed that Young's Modulus decreases as hinges were extended into the models. That is, the models are significantly more flexible as hinges were merged into the models.

The three model behaviors in the stretch test show two distinct regimes for typical solids: linear elastic regime, and the non-linear elastic regime after the yielding point. The kirigami models show a significantly higher failure stress, and a longer non-linear elastic regime before failure than a typical PLA block material. Hinges and cuts in the kirigami models behave as springs, which provide damping to the entire model when forces are applied. The damping properties make the kirigami models stronger than normal materials. Impregnating an ABS sheet with silicone, impacted stretching performance of the sheet as a whole by turning discrete bending moments into indeterminate, sheet-wide stresses. The integration of silicone into the plastic assembly may have distributed tension more evenly than the plastic sheet on its own, while also increasing resistance as a whole. In this way, the ABS plastic components function in a way analogous to a skeletal system, while the silicone acts

as the 'muscles' and membranes that stretch with and stabilize the underlying plastic structure.
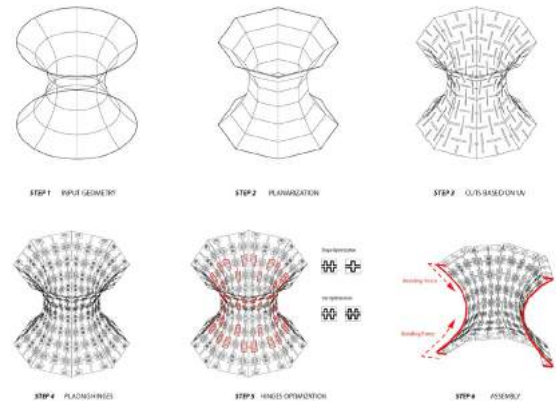

Figure 15. Process of embedding kirigami pattern onto a hyperbolic surface.

5 CONCLUSION

The goal for ColorFolds was to design deployable and scalable structures that respond in a controlled, kinetic mode to contextual feedback. We have established an adequate pipeline for simulating our kirigami structures in order to understand their behaviors on a global scale for larger architectural assemblies. With the pipeline we established, we can utilize the robustness of finite element analysis software ANSYS and the simplicity of physical simulation software Unity to achieve both quantitative and qualitative feedback. The pipeline can also be served as a cooperative platform to integrate both designers and engineers. Our next step is to inform generative design iterations with quantitative data from larger scale simulations in order to optimize and strategically design kirigami cuts and folds while addressing fabrication constraints and desired global curvature. Informed by our research described herein, our current investigations address 2D flat sheet to 3D form.
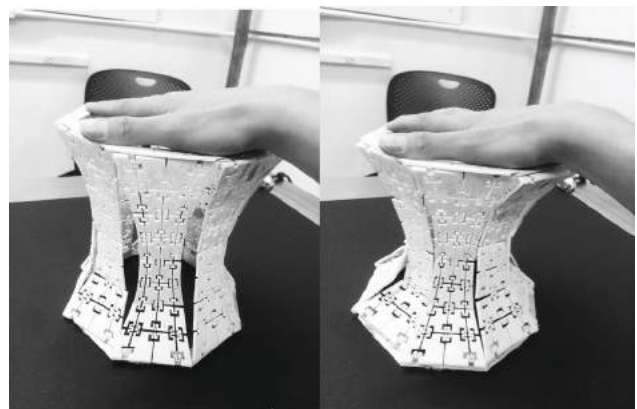

Figure 16. Physical prototype demonstrating that cuts and hinges allow the auxetic hyperbolic surface to be stretched and compressed to eliminate tolerance.

A developable hyperboloid shape was used as an input to test the feasibility of the computational model and the design-simulation-analysis loop, as theoretically demonstrated in Figure 15, and physically demonstrated in Figure 16. The given hyperbolic NURBS (Non-uniform rational B-spline) surface was first approximated by the planarized NURBS with degree of one. The planarization guaranteed that no bending or stretching force was exerted on the hinges. The surface was then divided into six sub-surfaces based on UV subdivision, cuts were placed aligned with UV curves and hinges were placed on the intersection. Each sub-surface was flattened and printed with inextensible plastic material ABS. In Figure 16, the tolerance caused by flattening and assembly was eliminated by the auxetic property enabled by embedded cuts and hinges.

Another immediate goal is the development of a simulation environment that is fully integrated with Rhino Grasshopper. Following this, large-scale physical prototypes can be produced using the programmable matrix, and optimized for different stretching behaviors in discrete zones. This matrix has the potential to become a deployable, 'adaptive' building material capable of optimized and differentiated expansion and contraction based on the properties of each unit. This improved process will greatly benefit large-scale applications that incorporate programmable material composites capable of controlled, elastic response to stimuli. Our ultimate goal is to generate a fluid and intuitive computational pipeline to facilitate a design process that is enmeshed with material and geometric feedback. Our work follows the concept of "Interact Locally, Fold Globally,"

necessary for deployable and scalable architectures. Using mathematical modeling, architectural elements, simulation, design computation, and controlled elastic response, this work showcases new techniques, algorithms, and processes for the assembly of open, deployable material systems and architectural surface assemblies.

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